Data Analysis

Hypothesis Testing and Confidence Intervals

Konstantinos Koutsompinas

This assignment consists of two parts. The first part requires to solve the solved exercises given in the slides. The second part requires your attempt.

Part I, duplicate the exercises presented in slides

**EXERCISE 1**

***Example 1***

The number of viewers for MEGA follow Normal distribution ~ N(150000,25000)

i) One day we count viewers X=185000 so:

H0: μ = 150000

Η1: μ > 150000

ε.σ. 5%

We calculate Q :

X < Q so we **accept H0**

ii) We calculate the average viewers in a 5 day span and we get 175000

The Average of the viewings follows normal distribution so:

H0: μ = 150000

Η1: μ > 150000

ε.σ. 5%

We calculate Q:

X > Q so we **reject H0**

***Example 2***

We have a milk store that believes that 25% of the milk cartons are not sold . In a sample of 300 cartons, 61 were not sold. Question : is the average=25% **overestimated ?**

We know that the analogy of the sold/unsold cartons , follows the distribution

so:

1. H0: p=0.25
2. H1: p<0.25
3. ε.σ.: 5%

We calculate Q :

X = 61/300 = 0.203 < Q so we **reject H0**

***Example 3***

A restaurant has the following number of customers for 7 Mondays: (n=7) 16 13 21 19 12 18 14

The restaurant will remain closed on Mondays if the average is below 20.

So we have:

1. H0: μ=20
2. H1: μ<20
3. ε.σ.: 5%

we calculate

-3.05 < Q so we **reject H0**

**EXERCISE 2**

To test whether people have various diseases such as tuberculosis or cancer, a preliminary test is made of the whole population over some specific age. State clear benefits of correct diagnosis and possible errors.

We have:

|  |  |  |
| --- | --- | --- |
|  | **Sick Person** | **Healthy Person** |
| **Test Positive** | True Positive | False Positive |
| **Test Negative** | False Negative | True Negative |

We can see that the errors are of 2 types :

* **False Negative:** The Person is sick but the test shows he is not. That error is of **major impact.** The person needs to start treatment immediately , however he is informed that he is healthy , and that could be a lethal error.
* **False Positive:** The person is not sick but is diagnosed as sick. In most cases this error is not as significant as the 1st one , since the person will most likely be subject to other tests after being diagnosed as sick , so the error will be found.

**EXERCISE 3**

***Example 1***

* Out of **100 cars** in Athens, **70** of them are European.
* Therefore, we approximately observe that **70%** of the cars are manufactured in Europe

We want to find a **95% confidence interval (CI)** for the proportion of cars manufactured in Europe.

**p = 0.7**, **n = 100** and we suppose that the analogy of the European Cars follows the distribution:

We calculate Q and R:

So a **95% confidence interval (CI) is [61%,79%]**

***Example 2***

* From a sample of **n = 190** workers, the **mean salary** is **€27,189**, and the **standard deviation of salaries** is **σ=€14,837**
* We want to find a **95% confidence interval (CI)** for the estimated mean of the sample.

We know that the mean of the sample follows the distribution:

We calculate Q and R:

So a **95% confidence interval (CI) is [22591, 31787]**

***Example 3***

* Consider a sample of **40 workers (n > 30)** from a population of **n = 190**, with a **mean salary** of **€27,189** *(and a population standard deviation of σ=€14,837*
* We want to find a **95% confidence interval (CI)** for the sample mean

We know that : follows t distribution with n-1 (39) degrees of freedom

So we have :

Part II

**EXERCISE 1**

Α gasoline additive is being tested to see whether it increases mileage. Twenty-five cars are supplied with 5 litres of gasoline and are run until the gasoline is exhausted. Αt the completion of the experiment the average mileage for each car is computed. Calculations with the data of this one experiment gave a mean of = 18.5 km per litre and a standard deviation of *s* = 2.2 km per litre for the 25 cars. Long-term experience with cars of the same kind that were used before when no additive was employed indicates that, µ = 18.0 and σ = 2.0 km per litre.

Assuming that the additive had no effect on mileage, answer the following questions:

* 1. Determine the probability accuracy of **as an estimate of µ. What is the actual accuracy? Is the sample estimate compatible with what was to be expected by theory?

The probability accuracy is determined using the standard error of the mean (SE):

The **actual accuracy** of the **sample mean** is given by the sample's standard error:

We compare the expected standard error (theoretical, based on σ\sigmaσ) with the actual standard error (based on sss). The **sample estimate is compatible with theory** if the difference between these values is not significant. Here, the difference is:

which is very small. Thus, the sample estimate is compatible with theoretical expectations.

* 1. Find how large an experiment should have been conducted if one wished to be certain with a probability of 0.95 that the estimate would not be in error by more than ½ km per litre.

We want :

So we need at least **62 cars**

* 1. Find a 95 percent confidence interval for µ. Does this interval actually contain µ?

So μ is in the confidence interval

1.4 Dropping the assumption that the additive had nο effect οn either the mean or variance, use Student's t variable to find a 95 percent confidence interval for µ.

**EXERCISE 2**

Answer the following questions:

2.1. Does the accuracy of the sample depend on the size of the population or on the size of the sample? On the size of the sample

2.2. If you double the sample size, how much more accurate will your sample become?

2.3. If you decide to accept the null hypothesis, can you be sure that it is really true?

No, you cannot be **completely sure** that the null hypothesis is true when you accept it. Accepting the null hypothesis means the sample data does not provide enough evidence to reject it, but this does not prove the null hypothesis. There is always a chance of committing a **Type II error** (failing to reject a false null hypothesis) The likelihood of this error is controlled by the significance level (σ.ε.).

2.4. If you decide to reject the null hypothesis, can you be sure that it is really false?

No, you cannot be **completely sure** that the null hypothesis is false when you reject it. Rejecting the null hypothesis means there is sufficient evidence to conclude that it is unlikely to be true, but there is always a chance of committing a **Type I error** (rejecting a true null hypothesis). The likelihood of this error is controlled by the significance level (σ.ε.).

2.5. Why is it necessary to choose a test statistic, whose distribution is known if the null hypothesis is true?

Choosing a test statistic with a known distribution under the null hypothesis allows you to:

1. Calculate probabilities (p-values) to assess the likelihood of observing the sample data under the null hypothesis.
2. Compare the test statistic to critical values to decide whether to reject the null hypothesis.

Without knowing the distribution, you cannot accurately determine the statistical significance of your results.

2.6. How much you have to increase the sample size in order to reduce the confidence interval width by one-half?

The width of the confidence interval is inversely proportional to the square root of the sample size. If you want to halve the width, the sample size must be increased by a factor of 4:

**EXERCISE 3**

The director of manufacturing at a clothing factory needs to determine whether a new machine is producing a particular type of cloth according to the manufacturer’s specifications, which indicate that the cloth should have a mean breaking strength of at least 70 pounds and a standard deviation of 3.5 pounds. The director is concerned that if the mean breaking strength is actually less than 70 pounds, the company will face too many lawsuits. A sample of 49 pieces of cloth reveals a sample mean of 69.3 pounds.

3.1 State the null and alternative hypotheses.

Η0: μ = 70

Η1: μ < 70

3.2 At the 0.05 level of significance, is there evidence that the mean breaking strength is less than 70 pounds?

We calculate

We calculate Q:

Since 69.3 > Q we **accept H0**

3.3 What will be your answer in 3.2 if you use a 0.01 level of significance?

We calculate Q:

Since 69.3 > Q we **accept H0**

3.4 What will be your answer in 3.2 if the standard deviation is 4.5 pounds?

We calculate

We calculate Q:

Since 69.3 > Q we **accept H0**